TONY CHADWICK PRIZE 1996 WINNING ESSAY

A THESIS:

That the development of a rational, innovative society, internally harmonious and tolerant of dissent, depends on understanding the original connection between mathematics and democracy.[1]

by Colin Hannaford

We have inherited our modern style of mathematical argument from the early Greeks. Their history is also our record of the early development of democracy.

I will argue that this style of mathematical argument was strongly influenced by social as well as intellectual interests; and that their mathematics was always constructed with a view to giving ordinary people insight into the methods of universal arguments.

The result was to favour of democracy. It gave the poor and uneducated examples of clear and strong arguments to counter the rhetoric and sophistry of the powerful and rich. It levelled the social arena.

In the form they developed every proof in mathematics is an explanation which everyone, at least notionally, can confirm. Because of its power thus to bring minds together, and unify people's endeavours, this is the style of mathematics we still use to-day.

If we show our young people that their mathematics education prepares them to expect and to demand democracy, in all their endeavours, political as well as scientific, we will achieve a momentous change in human consciousness and in politics.

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That mathematics education is fundamentally connected with democracy, and actively trains young people to accept and expect democratic rules, is usually not all difficult for one group to accept; but it is exceedingly difficult for another very much larger group. Basically, the problem we face is simply to connect the understanding of the two.

The group that finds it so easy to accept that mathematics is fundamentally democratic, is composed almost entirely of mathematics professionals: people who either depend on mathematics, or who create mathematics; and who therefore depend on these rules all their lives. They usually understand these facts at once.

This group, of course, is minuscule in terms of the world Its influence is certainly not so small, but it is also famous for its lack of interest in political questions. No professional mathematician has ever denied to me that they must work with other professionals democratically: I well remember one demanding, in fierce exasperation: 'How do people *imagine* that we would otherwise not cut each other's throats?' Undoubtedly some mathematicians feel that they can escape into mathematics from the world, and they fear that it may be contaminated by worldly affairs. From this, an even smaller, fraction, one should expect no help at all. From the rest there is usually benign, but detached, good will. They are, after all, *mathemati*cians, not *politi*cians. The *polis,* the people, must learn to manage their own affairs.

The other group is very large. It is indeed the *polis* itself: almost the rest of the educated world. This huge group has very serious interests in political questions, but it is also very hard for it to see any connection between mathematics and democracy. It has usually had an experience of mathematics very different from that of the mathematicians. Very different, and very discouraging.

Mathematicians, after all, *are* mathematicians, because they have learnt to use mathematics as a language. Incidentally, this means that so far from being the heirs of the aristocrats of the ancient Greeks, they are really the heirs of its working class, its *hoi polloi*. It is *their* language of argument that they have learnt, not the clever rhetoric and sophistry taught to rich young men by their private tutors, but the strong, clear style of argument developed as a reaction against such cleverness for everyone to use.

It may be of interest that it was approximately by this route that I began to understand that the very powerful effect that learning mathematics still has on democratic attitudes is quite unseen by those who are trained to think only classically.

In 1959 the British historian C P Snow, wrote of the two cultures of modern societies, of the scientists and non-scientists, into which all modern societies are dangerously divided, and the gulf of incomprehension between them. I had never realised how complete this gulf can be, until I casually pointed out one day to an historian friend how the democracy of the Greeks must have benefited from their development of mathematics. Essentially, I used the argument given above. My historian shook his head at me in wonderment. He understood me at once. 'But no-one has noticed that before,' he said.

No-one, in two and a half thousand years? No; no-one.

Another very important insight cost me rather more toil and trouble. For many years I taught mathematics in the belief that clever pupils are able to explain much better *because they are clever*.

This seems logical. But then finally I realised I had this the wrong way round. The clever are clever *because they can explain.* It is learning to explain, learning the argument first, which gives them their advantage.

And now I know that this is how the clever almost always learn. First they learn the argument. Generally we teach the others by persuading them to react to different patterns of symbols and shapes. They never learn the argument. 'It should look like this!' they tell each other privately. Of course we think they are learning like the clever ones. But they are not. They are just remembering patterns, which is how they 'should look.' Eventually they cannot remember enough of these patterns; and then they begin to fail. In fact we have allowed them to program themselves, as children under pressure almost always will, so that they must, inevitably, fail.

This is generally the very unpleasant, humiliating experience of mathematics which most of the larger group have had. It may also have had some good effect on them of course. They may recognise the power of general logical arguments. But usually they do not see these in mathematics. They have almost never recognised that mathematics is all general arguments. 2 + 2 = 4, is a general argument; but not to them. To them it is a pattern that you have to know. So

we fail in this way too. It is as if they know that water will keep them clean: but we have taught them to fear and dread the rivers and the sea, and they will only bathe themselves in meagre streams.

Mathematics has been imposed on this group mainly as an authoritarian science. It has never offered them room for discussion, or freedom to dissent, and almost certainly it has shown little patience with error. They know that all of these are essential for democracy. They will also almost certainly believe that mathematics is *only about numbers:* and numbers, they will say, cannot encompass all that politics entails.

It is therefore completely unsurprising that this large group - it is, in fact, most people - will generally regard a proposal of a connection between mathematics and politics as the kernel of a plan to suppress dissent and extinguish democracy. They cannot conceive of it in any other way.

They will also not credit the fact that dissent is very common in learning mathematics. Usually we don't call it that. Usually we simply say that the class does not understand. Of course there is the possibility that the teacher is really wrong, and that his argument is actually false. This would be a real dissent over fact, and is more difficult to deal with. But let us examine the more common possibility first. The teacher is right, but he cannot persuade his pupils. They dissent from him over his belief. What does he do?

At this point a very important lesson in democratic manners is to be learnt; and it is learnt in modern classrooms, almost invariably. In the bad old days the teacher might simply flog his pupils to assent. This would produce no democratic manners at all. Democracy is taught, instead, when the teacher accepts the disagreement, and tries to present his argument in a way that the class can accept. It sometimes happens - it has certainly often happened to me - that some of the dissent is really about the facts and is correct. The teacher can be wrong Then the class corrects the teacher. This is democracy at its best.

Ideally there is always time to achieve results like these. In practice there is never sufficient time and there are other formidable obstacles too. (You really didn't think this would be easy?) In recent years several important academic studies have been made in several countries which have sought to establish whether mathematics has any political importance, and whether social pressures affect the way in which mathematics is taught.

In both of these cases the conclusions of the studies that I know about are seriously at variance with my own, and I should at least indicate where the variation lies.

In the first case (typified by a recent lengthy Danish study: in fact I do not believe it is finished) it was decided that mathematics itself is politically neutral, but that some mathematical knowledge is essential in 'empowering' democratic participation. I have tried to show already why I think this fundamentally misrepresents mathematics. I say that mathematics is strongly active politically. It supports the mentality which wants democracy. In the second case (typified by a paper written in 1986 by Dr Paul Ernest, now the chairman of the Philosophy of Mathematical Education group), it has been the effects of social and political decisions *on* mathematics teaching which received attention, rather than the effects of mathematics teaching *on politics.* Obviously, again I believe this is the wrong way round.

And, finally, in the past four years it has been very interesting for me to discover how very rare is real trust in one democratic process. Many people fear democracy very deeply. What they really seem fear is the displacement of the decision-making of their society from the extreme end of the bell-curve of intelligence distribution to somewhere near the middle. This seems to them as disastrous as it did to Socrates and to his band of young men, several of whom were very notable in attacking and betraying Athenian democracy. They wanted only the best to rule them. The best they got, the rule of the Thirty, in 404 BC, turned out to be plundering killers.

History has taught us that the bell-curve has two extremes. When a polity is constructed to give an advantage to one extreme, either may seize this advantage. A society like this may be ruled by saintly ascetics for a while, but then by brutes and dregs. To be ruled from the middle of the bell-curve is, overall, much safer. (And notice, please, that your understanding of mathematics has helped to make the political argument clear!)

There is no need to prove to anyone that mathematics is important and powerful. The problem is to focus attention on *the methods* which have been employed in creating the results, rather than to focus only on the results themselves. The results are certainly politically neutral. They may be used by machines. The methods are not neutral at all.

I have found that one of the most effective ways of making this clear has been to show the similarities *of form* of the teaching of mathematics (proper teaching: rather than terrorising one's pupils to assent) and democratic politics.

Here are the similarities. I wrote them out originally for a student of mine in China:

In mathematics classes:	**In democracies:**
1. Teachers must treat their pupils (and the pupils must learn to treat each other) as equals;	**1. Political leaders must treat their voters (and the voters must learn to treat each other) as equals;**
2. Their arguments must be completely and clearly explained;	**2. Their arguments must be completely and clearly explained;**
3. An argument is only satisfactory when the majority of pupils can and do accept it.	**3. A policy is only satisfactory when the majority voters can and do accept it.**

And there is finally one other fact of common experience which I find helps in convincing people of the real value of mathematics. Anyone who has ever seen the joy on the face of a child experiencing the power of its mind in understanding any piece of mathematics will know what this is.

Mathematics has the ability to help complete a person's spiritual formation.
You may find this the strangest suggestion I could make about teaching mathematics: or you may understand at once what I mean. Here, I think, is the true crux of the new understanding of mathematics that we must develop; and we need it soon.

Mathematics has this ability because it can help young minds to realise that they can share in the management of their world. In a sense, they can share in its creation. They can share in its improvement, or share in its destruction. They are people, not helpless, but with power.

I think that we neglect this dimension at very great loss to our own peace of mind and to the peace of our hearts. We are taught, very blindly I think, to regard mathematics as the antithesis of humanity. We may learn it in this world, but has nothing essentially to do with this world. I think this is very wrong. The great pre-Christian philosopher Plato saw mathematics as a way to perfecting the soul's vision of reality. So do I understand mathematics.

In this vision, minds speak to minds with honesty, without jealousy, without rancour, without envy; they understand their society as sharing one mind; and as that one mind as being all that evolution can achieve. We cannot have sane societies which are in harmony with the world unless we teach young people to understand this unity of minds.

Mathematics is all the proof we need that it is possible.

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1 A first paper explaining these ideas was published in Germany in 1993 by the Gesellschaft für Didaktik der Mathematik, the National Association of Mathematics Teachers. It was reported in this year too in France by the Association des Professors de Mathématiques de L'Enseignment Public. In 1994 I was invited to lecture in Germany for the Landeszentrale für politische Bildung of Baden-Württemberg. In 1995 Mme Edith Cresson, the European Commissioner for Education, asked me and my associates to prepare a submission for the pan-European Comenius project, which will train the future teachers of Europe.