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THE LIAR PARADOX OUTSIDE-IN

To classify sentences like “This proposition is false” as “having no truth value” or as “non-propositions” is generally considered as being an ineffective strategy to solve the Liar paradox because, among other reasons, the paradox only reappears in another, strengthened version. I shall present and defend in this paper an outline of a “dissolution” of the Liar paradox that pretends to overcome those objections and that does just this: to classify the Liar sentence as a non-proposition based on a principled argument. I will show that it is immune to the Strengthened Liar version and discuss a major objection.

Liar and Strengthened Liar

The basic version of the Liar sentence comes in many forms, depending on what one prefers to take as truth bearers, e.g. as an utterance token: "I am a liar" or as a proposition: (L) "This proposition is false".

Typically the analysis for the basic version that leads to a paradox runs like this:

- L1. (L) is a proposition.
- L2. (L) is either false or true (assumption of bivalence)
- L3. Imagine that (L) is true. Then what the proposition says is the case. The proposition says it is false. Hence it is false. Which is a contradiction.
- L4. Imagine that (L) is false. Then what the proposition says is false and the contrary must be true. The proposition says it is false. Hence it is true. Which is a contradiction.
- L5. Hence if (L) is true it is false, and if (L) is false it is true

The more intricate version of the Liar paradox is the “Strengthened Liar” (e.g. Rieger, 2001, p.3 or Kirkham, 1992, p.293). Typically solutions that try to solve the paradox by giving the Liar sentence a special semantic status, e.g. “neither true nor false” or “has no truth value (falls into a truth value gap)” or “meaningless”, are unable to deal with it. As an example, the argument that leads to a new paradox for the sentence “This proposition is either false or has no truth value” (SL) is:

(PSL):

- SL1.) SL is either true, false or has no truth value.
- SL2.) If SL is true it is what it says, namely either false or it has no truth value. A contradiction.
- SL3.) If SL is false, hence it is not either false nor has no truth value. This means it is true. A contradiction.
- SL4.) If it has no truth value, it is exactly what it says, but then it is true. A contradiction.
- SL5.) Hence, it can neither be true, nor false, nor without a truth value. A contradiction to the assumption in SL1.

Similarly, we can build strengthened versions for the other-above mentioned expressions. In whatever terms we settle the semantic category of the Liar sentence, the paradox reappears !

Adequacy criteria

It seems we can safely say that no solution of the Liar paradox is yet universally accepted. Taking two very influential proposals as examples, we may understand why this is the case.

Tarski (1936; see also Kirkham, 1992, pp.278-280) distinguishes a hierarchy of formal object- and meta-languages. He stipulates that each level must not contain truth predicates to express truths in the same or higher levels, but only that of lower levels. Hence Tarski solves the paradox simply by blocking the basic Liar argument in step L1, because the Liar expression is not accepted as a well-formed proposition. It is generally recognized, even by Tarski himself, that this solution is not appropriate for natural languages (see Burge, 1979, p.171). The concept of truth in natural languages simply is not hierarchical. Tarski's solution also seems extensionally inadequate, because the non-paradoxical truth-teller expression "This proposition is true" is also excluded. In general, Tarski's stratification is not naturally motivated and seems ad-hoc.

Kripke (1975; see also Kirkham, 1992, pp.282-294) proposes a solution for natural language propositions containing their own truth predicate. The main idea is to construct a maximal set of true and false statements in a progressive way, starting from statements that do not contain semantic ascriptions. As a result, and once the process stabilises at a "fixed point", where no new truths or falsehoods are effectively added, the Liar remains a sentence without truth value assignment. It is generally argued that this introduces a truth value gap for Liar sentences and that the solution is hence vulnerable to the Strengthened Liar version (Burge, 1979, p.173). His solution may be intuitively more appealing than Tarski's; however, it also excludes the truth-teller sentence (though it can include it in a purely ad-hoc fashion into the set of truths or falsehoods) and hence his solution cannot be considered as extensionally adequate.

The upshot is that proposed solutions typically are unable to fulfil satisfactorily all criteria of adequacy: 1. It should avoid ad-hoc postulations. 2. It should not dismiss non-paradoxical types of sentences (extensional adequacy) 3. A solution should apply to the different variants of the paradox, including the Strengthened Liar. 4. It should conserve our intuitions about truth and validity of arguments (intuitive adequacy). Let me add to this list from Kirkham (1992, p.272) another one: 5. It should be applicable to different truth-bearer types.

Truth value gaps and third truth values

It is generally argued that to deny the Liar sentence the status of a proper statement or proposition, to say it is a non-proposition or non-statement, is ineffective because this implies vulnerability to the Strengthened Liar. I will show that if the notion of "non-proposition" is understood in a certain way the Strengthened Liar argument fails.

Burge says, for example, that “*a non-statement is obviously and categorically not true.*” (1979, p.178). I think that Burge does not give the necessary importance to a certain ambiguity about what it means to “settle a semantic category” for something. The “is not true” (the same applies for “is neither true nor false”, “has no truth value”, etc.), on which a Strengthened Liar can be constructed, can be read in two ways:

a) “is not true” as not being *true*, but maybe *false*, or maybe having *another* truth value; maybe it just has an unsettled, undefined, undetermined truth value, but in any case it is presupposed that the expression is asserting something (truly, falsely, X-ly,...), and *could* have a truth value, such that it makes sense to take the expression as having an assertoric character.

b) “is not true” in the sense that truth values are not meaningfully predicable of the subject term, because of a clear conceptual incompatibility. E.g. certainly a fridge is an “un-semantic” thing and it would not occur to us to say that “The fridge is not true”. One may respond that a fridge “does not have the property of having the truth value ‘true’”. I respond that a fridge does not have *any* truth value in the strong sense of conceptual incompatibility e.g. between a fridge and the truth predicate.

If we ignore the difference between a) and b) it is like ignoring the difference between “This car is not red” and “Time is not red” and certainly there is a difference. You can deny a *specific colour* or you can deny the *conceptual applicability of the colour predicate as such* to the subject term. Failure “to have the property of being red” can then take place on two levels.

As an upshot, something “un-semantic” fails to have a truth value or fails to be true in the strong sense of b). Now, the argument (PSL) that leads to the claim that “This sentence has no truth value” is paradoxical is not valid assuming that “has no truth value” is to be understood as “being un-semantic” as in case b). Once you assume that SL is not a thing with an assertoric character, step SL4 is not valid. You cannot, as is done in this step, extract from a thing we just assumed to be non-assertoric, anything assertoric (“what it says”).

On the other hand, to introduce a genuine third truth value, let's say “frue”, on the same level as “true” and “false” is not effective because this implies that SL is still a semantic thing (it *asserts* something that is either true, false or frue). In this case we could extract from the SL “what it says” and the Strengthened Liar argument is effective.

Therefore, we must conclude that to classify SL as “un-semantic” in the sense of b) is an effective strategy to block the Liar that is also immune to the Strengthened Liar. Now that we have tried to settle the claim that *if* the Liar were un-semantic we could block the paradox, we need to construct an argument that *it is indeed* un-semantic.

The Liar sentence as a non-proposition

Utterances

The idea is to analyse the Liar sentence “I am a liar” *outside in* (from the world – in the form of matters of fact – to words). This means, let’s establish the subject matter the sentence is apparently about, identify exhaustively the relevant cases in terms of possible matters of fact

related to the subject matter and see the consequences in terms of possible assertoric utterances that can arise in each case.

I will show that the Liar sentence is not among all possible assertoric utterances. This does not mean there are no such utterances, there obviously are, but there are not utterances *qua assertoric speech acts*, which is a necessary condition to consider them as truth bearers and hence to class them in the semantic category "semantic", as just defined. The sentence may arise spoken with an other than assertoric purpose. Also it may arise as a failed assertoric act, but this implies the person does not know exactly what she is talking about and does not apply correctly the concepts contained in the sentence; hence it is to be taken as un-semantic nonsense.

Assume there are only liars and truth-sayers (otherwise the paradox would not work in the first place). Then, given a certain person, there are only two possibilities (as matters of fact): either he is a liar or a truth-sayer. Now, the Liar sentence has a clearly identifiable subject matter: the condition of oneself as a liar or a truth-sayer. Let's take case 1: a liar says something about his condition as a liar or truth sayer. He would say, falsely, as a liar: "I am a truth sayer". Case 2: the truth-sayer would say about his condition, truly, as a truth-sayer: "I am a truth-sayer".

Both cases exhaust all the possible cases with regard to assertoric utterance about the given subject matter. It is simply not possible that "I am a liar" arises *as an assertoric speech act*. It is an un-semantic utterance and hence the Strengthened Liar is blocked, as argued in the preceding section. We would be making a category mistake to take it as something that asserts. We cannot extract from the un-semantic word form "I am a liar" the "what it says (assertorically)" and infer a contradiction.

Propositions

By analogy we can show that "This proposition is false" is not among all possible propositions.

Let's assume here, following the general use of the notion, that a proposition is an abstract truth bearer and that propositions "correspond" to matters of fact (which hold or could hold). To account for bivalence, we introduce two assignment functions from matters of fact to propositions:

- 1.) the "true proposition function" (TP): assigns the (true) proposition $P(M)$ to a certain matter of fact called M
- 2.) the "false proposition function" (FP): assigns the (false) proposition $\text{not}(P(M))$ to a certain matter of fact called M .

Assume that the truth or falsity of a proposition counts itself as a matter of fact. Also let's say that a proposition is "generated" by TP (or FP) if it stems from an assignment via TP (or FP).

Now, let be M the matter of fact about the truth or falsity of a proposition P that expresses that very same matter of fact M . There are two cases:

- (a) P is "generated" by TP (is true, as a matter of fact), hence the proposition assigned to M is $P(M)$. Now by definition of TP it is a matter of fact that any proposition generated by TP is true, hence $P(M)$ is true as a matter of fact. But this is the

matter of fact M (that P(M) is true). Hence $P(M) = P(P(M) \text{ is true})$. But as we deal with P(M) expressed in P(M) we can say: P(M): P(P(M) is true) or in word form: *"This proposition is true"*.

(b) P is generated by FP (is false, as a matter of fact), hence the proposition is: not(P(M)). Now by definition of FP it is a matter of fact that any proposition generated by FG is false, hence (not(P(M)) is false), which is the very matter of fact M (that not(P(M)) is false). Now the output from FG was not(P(M)) and if we replace M by (not(P(M)) is false), we get: not(P(not(P(M)) is false)) = P(not(P(M)) is true), due to bivalence, and again, as we deal with not(P(M)) expressed in not(P(M)) we can say: not(P(M)): (not(P(M)) is true) or in word form: *"This proposition is true"*.

Again we have exhausted all possible matters of fact about the relevant subject matter. In analogy to the version with utterances as truth bearers we have shown that "This proposition is true" can be generated (in two cases); however, "This proposition is false" cannot be generated. Hence "This proposition is false" is not in the relevant set of possible propositions. The conclusion is the Liar sentence cannot assert anything (in bivalent classical logic) and hence is un-semantic.

I have argued that to classify a word form as "un-semantic" is effective against PSL and I have tried to argue plausibly that the Liar sentences are un-semantic. If the reasoning is accepted, then the Liar paradox seems to be resolved.

An objection

It looks odd that just by replacing "true" in the sentence "This proposition is true" by "false" we cross the border from proposition to non-proposition.

"Semanticality" (the property of being semantic or un-semantic, as defined in this paper) should be preserved when we replace one possible value of a predicate for another. E.g. "This car has the color X" is for *all* color values X a proper proposition. The proposed solution implies that semanticality is not preserved for certain sentences containing a truth predicate when changing "true" for "false".

An extensionally adequate solution to the Liar should isolate *exactly* the genuinely paradoxical sentences based on a principled selection process. The proposed method is principled in the sense that it makes some general intuitive assumptions (about propositions corresponding to facts, bivalence), and the different classification of the Liar and of the truth-teller sentences is not necessarily a predictable outcome, hence not ad-hoc. The implications of a solution to the Liar paradox that fulfils the adequacy criteria set out at the beginning, especially if it is extensionally adequate, may then be taken to be potentially conceptually revealing.

How do we explain that "This fridge is not true" is nonsense? Conceptually a fridge is such that truth/falsity is just not the thing that we can predicate of a fridge. What those things like fridges are conceptually we learn by using those concepts. And they are not "semantic", so they have no "truth value gaps" and neither shall we say they are "not true". It seems that somehow into the concept of truth/falsity as we use it, the idea is built-in that falsehood does not reveal itself. This leads a conceptual incompatibility not for the whole truth predicate with all its values, but - in the

case of circular structures typical of Liar sentences or groups of sentences - to incompatibilities with one of the values ("false").

Someone sceptical about compositionality of language, i.e. someone rejecting that sentence meanings arise from word meanings plus syntax, may not even conceive a problem with this asymmetry. It is enough to have shown that as long as we stick to matters of fact, paradoxical sentences simply cannot arise. Those sentences are the result of fact-detached compositions of word forms. Instead of seeing a problem they could say: the worse for the principle of compositionality. Of course, I don't think that compositionality needs to be rejected entirely. However, defenders of this principle cannot deny that linguistic combinatory possibilities outstrip possible facts. They already need to give in when there are conceptual incompatibilities between predicate and subject.

Now it turns out that the Liar reveals a more intricate conceptual incompatibility between a specific subject (word form itself) and predicate (is false/true), than examples like "My green dream sleeps furiously". In any case, what makes a sentence un-semantic are such conceptual incompatibilities.

A final remark, and to anticipate another possible objection: Kripke showed that paradoxicality is not a question intrinsic to the meaning or syntax of a *single* sentence (1975, p.692). Sometimes it is an empirical phenomenon that a sentence in isolation is non-paradoxical, but taken together with another one, which is also not paradoxical in isolation, becomes paradoxical. The following pair of utterances e.g. is paradoxical:

A: "All that B says is true".

B: "All that A says is false".

The outside-in method works also for such circular groups of utterances and propositions. We can show with the same method that such groups cannot arise from matters of fact.

Conclusion

It has been argued that the Liar sentence is not among all possible propositions. This does not mean that it has a truth value gap or a third truth value type. Rather the Liar sentence is semantically inert, equivalent to a line of a poem that does not say anything declaratively. Therefore, the Strengthened Liar argument is not effective against a strategy to settle the Liar sentence as a non-proposition.

If the line of reasoning is correct, then in the case of using the truth predicate, linguistic combinatory possibilities outstrip possible facts even more than in the case of ordinary predicates, because simply by changing "false" for "true" in "This proposition is true" we move from a proper proposition to an un-semantic word form.

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